AquaMOOSE 3D: A Constructionist Approach to Math Learning Motivated by Artistic Expression

Thesis Proposal
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Introduction

Teachers often explain students’ performance in school classes in terms of their interest or lack thereof. *Fred is just not interested in math. Sarah is interested in writing, so she is more engaged in classes where she can write essays. Jack was really interested in that project and did an excellent job on it.* Research has shown that student interest is positively related to various measures of learning such as attention, comprehension, and thinking (Schiefele 1991; Tobias 1994). Additionally, it has been shown that students’ interest in academics declines significantly with age, especially in the areas of math and science (Wigfield and Eccles 1992; Kahle, Parker et al. 1993; Wigfield 1994).

Technology is becoming ubiquitous in schools. However, researchers and teachers are still searching for meaningful ways to use these new tools to help students learn. Many strategies and software tools have been devised over the last few decades with varying degrees of success and impact. One approach to creating successful interventions is by designing systems that look at the entire learning environment instead of simply the technology. Socio-technical systems are specifically designed to integrate both the technical system and the social system in the environment (Ketchum and Trist 1992). Interactions between participants, peers, mentors, teachers, and the technology are all equally important to the success of the intervention.

As the primary focus of my research, I propose a socio-technical system that explores one approach to positively affecting students’ interest in academics: by combining two diverse subjects in the same learning intervention, students can leverage their interest in one academic area to gain a better appreciation of (and become more interested in) the other area. I plan to instantiate this socio-technical system in an after school program that focuses on creating mathematical art with a new software tool called AquaMOOSE 3D. This software tool, developed over the last six years using an iterative design process, provides students with a way to create interesting virtual artifacts, primarily through the use of “math trails” that represent sets of 3D parametric equations.

Using this socio-technical system, I plan to answer two research questions:

1) How can we use construction in an engaging artistic domain to improve learner interest in academic subjects, specifically mathematics?
2) What different trajectories do learners utilize in a constructionist learning environment that invites them to learn an academic subject by engaging in an artistic endeavor, and how do differences in prior interests affect those trajectories?

My plan for completing this research includes designing the AquaMOOSE socio-technical system, integrating the system into a 12-week after school program, and collecting data during the after school program to describe participants’ experiences and how those experiences affect participants’ interest levels in math and art. This document explains in further detail how each of these steps will be accomplished.
AquaMOOSE Socio-Technical System

The design of the AquaMOOSE socio-technical system has been greatly influenced by Seymour Papert’s notion of constructionism (Papert 1980; Papert 1991). Constructionism advocates that people learn better by building personally meaningful artifacts and sharing them with others. However, despite numerous examples of constructionist learning environments that have been studied recently, our understanding of how students become engaged and learn in a constructionist environment is incomplete. A secondary focus of this research is to further describe the constructionist learning process, looking specifically at how prior interest affects the way students approach and use the AquaMOOSE technology to create personally meaningful artifacts.

One typical trajectory we have observed while evaluating the AquaMOOSE software begins with students typing in equations with random math functions. Based on the results of their initial experimentation, students continue adding random elements to their mathematical equations. This phase of the learning process is purely exploratory. At some point during the exploration process, the student generates a goal, usually one dealing with some visual aspect of their current artifact. Once this goal has been established, the student works to refine the math instead of randomly trying different types of functions. Through continued refinement and feedback from mentors and peers, students work toward achieving a specific goal for each artifact.

Students with different interests approach the AquaMOOSE system in different ways, however. In informal testing, we have seen that users with a basic interest in art tend to explore more and refine less, generating a goal later in the process. Users with a basic interest in math tend to explore less and refine more, creating smaller and more math-oriented goals early in the learning process. In this research, I will collect data to further describe variations in learning trajectories between students with varied prior interests.

Over the last six years, the AquaMOOSE 3D software has been developed using an iterative design process that involved many user trials (Elliott, Adams et al. 2002; Elliott and Bruckman 2002). The software consists primarily of a 3D graphical environment where the user is represented by a fish avatar. The avatar’s motion can be specified mathematically by entering a set of parametric equations. Once users have created a mathematical “trail” by typing in parametric equations, they can manipulate the appearance and animation characteristics of that artifact (see Figure 1). The design and evaluation of the AquaMOOSE software are described in further detail in Appendices A and B respectively. Throughout our iterative design process, we have continued to refine the potential combinations of math and art in the software.
Figure 1: See Appendices A and B for more details about the software tool

We have also learned from students’ unexpected uses of the software during our user trials. Most notably, the idea of providing explicit support for artistic expression through math came from a student’s experience in our first large-scale study. This student, whose experience is described in greater detail in Appendix B, used the software to create complex artifacts that used by-products of our calculations to create such artistic effects as “barbed wire” and “solid tunnels.” Insights such as this led us to believe that explicitly designing synergy between math and art into the software could impact students’ level of interest.

In the next section of this document, background literature and related work are described. The third section details the proposed study and expected contributions of this research. The first appendix provides a description of the design process we have followed over the last 6 years to develop the AquaMOOSE software. The second appendix presents the multiple rounds of formative evaluation we have completed over the last several years.
Background Concepts

The AquaMOOSE socio-technical system is designed to positively affect students’ interest in math and art by providing a constructionist learning environment where students create and share works of mathematical art. This section provides background information about each of the components of this system.

Interest and Learning

The term “interest” is often used interchangeably with the term “motivation.” In the motivation literature, interest is usually described as one factor that influences motivation. Motivation is defined as “the process whereby goal-directed activity is instigated and sustained” (Pintrich and Schunk 1996). When laypeople are asked to define motivation, however, they often describe it in terms of a person’s interest in performing some task or activity. This definition of motivation emphasizes the enjoyment a person gains from doing a task or their subjective interest in the content of a task (Wigfield and Eccles 1992). For the purposes of this research, we refer to interest as an internal state that causally impacts students’ attention, learning, thinking, and performance. It is this construct that we attempt to influence with the AquaMOOSE socio-technical system.

During extended learning experiences, interest has been shown to wax and wane based on the activities students are engaged in at a particular stage of the learning process (Joseph and Nacu 2003). Joseph and Nacu propose that learning environments should incorporate strategies for sustaining interest throughout the learning intervention. Specifically, they suggest that support should be addressed during both the initiation and maintenance stages. Initiation strategies involve introducing activities that are designed to direct students’ attention to a particular topic that might be interesting. Maintenance strategies involve using context-based supports such as deadlines or social motivators to help boost interest levels during the times when interest is expected to wane. In my proposed research, we have designed activities specifically to address these concerns. For example, throughout our system we provide participants with “challenges” that are intended to help initiate interest in specific concepts. We also include specific deadlines, such as a critique session, to help motivate students when we expect their interest to be waning.

Socio-technical System

Socio-technical system theory proposes that organizations are comprised of two independent, but linked, systems: a technical system and a social system. The technical system involves technology and processes, while the social system involves people and social practices (Ketchum and Trist 1992). The technical system includes computers, computer software, and other inanimate objects such as pencils and calculators. The social system includes interdependencies between the actions of multiple persons,
including “communication and cooperation structures, formal organizational structures, personal expectations and interest or qualifications” (Herrmann and Loser 1999).

We have seen in our past work that deploying the AquaMOOSE software with a stronger emphasis on the technical system than the social system often does not achieve the expected results (see Appendix B for further discussion). For this proposed research, we have developed an educational intervention that emphasizes both the technical system and the social system. The AquaMOOSE software remains a key component to the goals of this research, but the curriculum design, social interactions, feedback mechanisms, and audience aspects are also critical to the effectiveness of the intervention. Each of these components has been carefully designed and is an integral part of the AquaMOOSE socio-technical system.

**Constructionism**

In traditional classrooms, students are expected to receive information relayed by the teacher, absorb it, and come to understand it in the same way that the teacher understands it. This is called the transmission model of learning. Jean Piaget proposed an alternative model of learning that he called constructivism. Constructivism argues that people actively construct knowledge based not only on the information they receive, but also on their context and understanding of the world (Piaget 1972). Constructionism, which is an extension of Piaget’s constructivism, theorizes that people learn better through building personally meaningful artifacts and sharing them with others (Papert 1991; Bruckman 1998). Constructionism is the main educational philosophy underlying the design of the AquaMOOSE system.

One aspect of a constructionist learning environment that plays a critical role is audience. Students learn more effectively when they are creating artifacts that will be viewed and commented on by other people. The AquaMOOSE system is designed to support users sharing and discussing each other’s math creations through the technological system and the social system. Participants can build objects in their “private” space within the software, and then simply move them to their “public” space when they feel comfortable sharing them with the rest of the AquaMOOSE online community. Participants can also utilize the social aspects of the system, such as advisory sessions, critiques, and physical collocation, to get feedback from peers, teachers, and mentors at various stages of their learning process. The final objective of the proposed AquaMOOSE after school program is for participants to create a portfolio of objects that will be presented to a local audience of friends and family at the end of the workshop and optionally displayed on a web page hosted by Georgia Tech.

Constructionism also emphasizes the need for learning activities to incorporate both personal and epistemological connections (Resnick, Bruckman et al. 1996). The AquaMOOSE system facilitates personal connections by emphasizing the visual appeal of the mathematical trails and animations. Participants in the AquaMOOSE system are often familiar with similar graphical effects from other settings, such as video games and
movies, and are engaged by the possibility of creating such effects. Complex combinations of mathematical concepts, used in novel ways, lay the foundation for students’ epistemological connections. Students are not often taught in school that math can be used to make intricate graphical designs, and this environment gives them the ability to freely explore those possibilities by building on their prior math knowledge. By participating in the AquaMOOSE system, students can build interesting visual objects that combine their imagination with important mathematical concepts to create beautiful and elegant works of personally meaningful mathematical art.

Cognitive Apprenticeship

Cognitive apprenticeship describes an approach to designing educational interventions that attempts to translate the process of traditional apprenticeship-based learning from physical practices, such as blacksmithing or tailoring, to cognitive practices, such as reading and writing (Collins, Brown et al. 1989; Lave and Wenger 1991; Guzdial and Kehoe 1998). Cognitive apprenticeship incorporates many concepts from traditional apprenticeship, such as having expert practitioners modeling the target skill and providing fading support or scaffolding for learning the target skill.

One aspect of cognitive apprenticeship that plays an important role in the AquaMOOSE system is the ability for participants to observe other learners who are at varying stages of the learning process. According to Collins et al., “this encourages [learners] to view learning as an incrementally staged process, while providing them with concrete benchmarks for their own progress (Collins, Brown et al. 1989).” Through the community and discussion features in the AquaMOOSE software and the social feedback processes of the AquaMOOSE system, such as advisory sessions and critiques, students are able to see how other participants are progressing, and can easily build on the concepts other students incorporate into their creations.
Related Work

This section describes projects that have influenced the design and implementation of the AquaMOOSE socio-technical system.

Green Globs

The Green Globs software is a software tool that uses simple 2D graphics-based games to teach high school students about algebra. Students are presented with a screen containing several dots, or “globs”, that are randomly placed on a grid. Their goal is to make “shots” using polynomial algebraic equations that will hit as many of the globs as possible, and take out all of the globs in the fewest number of shots. The children using the Green Globs software were motivated by the game scenario and were led to explore many complex algebraic solutions that they would not have been exposed to otherwise (Dugdale 1982).

In designing the Green Globs software, Dugdale and colleagues made some interesting design decisions to provide an appropriate level of motivation for students to explore new types of algebraic concepts. One contribution was to devise a feedback system that gave students of various mathematical abilities an equal feeling of accomplishment and desire to improve their problem-solving strategies. Scoring in green globs was originally done by counting the number of shots it took a student to burst all 13 globs. The problem with this arrangement is that most students could burst all of the globs easily in 6 shots, but it took an extremely favorable layout of globs to do so in less than 4 shots. With such a small range of possible scores (4 to 6), there was little motivation for students to improve their approach to constructing equations. The scoring algorithm was then changed to provide more points for each consecutive glob hit with a single shot. So the first glob would score 1 point, the second 2, the third 4, the fourth 8, and so on. With the new scoring algorithm, Dugdale found students of varying math skill were equally interested in improving their equations to score more points.

Another contribution of the Green Globs project was Dugdale’s recognition and encouragement of creative exploration in academic tools. Students using Green Globs devised many creative methods for bursting large numbers of globs in a single shot. For example, students learned to use factors of the form \((x – a)\) along with large coefficients to create nearly vertical lines on the graph. Combining several of those factors allowed students to hit many of the globs with a single equation. This type of creative solution is something that students would not normally internalize based simply on classroom exposure. In the Green Globs environment, however, students were motivated to understand and use concepts that would otherwise be just another homework assignment. Green Globs was also designed to encourage students to explore algebra freely, without being constrained to the specific graphs or concepts they are studying in math class.
The Green Globs project inspired the creation of the AquaMOOSE game space called the Ring Game. In the Ring Game, students create a sequence of rings using mathematical equations and then challenge their friends to solve their ring puzzles. As in Green Globs, the students do not have to guess the exact math equations, since the rings have a large diameter and passing through any point within the ring is sufficient. However, this game turned out to be too difficult for students to use, due mainly to the added difficulty of the third dimension (Elliott, Adams et al. 2002). Given the limitations of the Ring Game, we have focused on different mechanisms to support the type of motivation that the competitive game provided in Green Globs.

Green Globs serves as a positive example of educational technology that increases student interest through creative exploration of complex mathematics. Students are allowed to try out different math concepts at their own pace in a playful setting. Even though the students are often motivated by competition to score more points, the system was designed so that students at various ability levels can improve and feel successful in their explorations. The AquaMOOSE socio-technical system aims to provide a similar opportunity for students to become more interested in math and art through the process of creating personally meaningful mathematical artworks.

**Instructional Software Design Project (ISDP)**

In the first instantiation of the Instructional Software Design Project, 4th grade students built LOGO programs to teach 3rd grade students about fractions (Harel and Papert 1990). In an extension of that work, 5th grade students designed software for 4th graders, and then became consultants for the 4th graders as they designed software for the 3rd grade students. In this second instantiation, the older students enjoyed taking on the role of consultants for their 4th grade peers. “It provided students with a different audience from the one they usually have. In regular school-like situations – by facing a teacher, who by definition seems to know everything – students might feel intimidated to announce their ideas or problems, discuss their theories and raise hypotheses (Kafai and Harel 1991).” In other words, the students were able to approach the learning task from a more playful and novel perspective that allowed them to leverage social interaction with their peers to shape and refine their understanding.

One benefit of consultation for learning is that it provides the students an opportunity to step back from the creation process and think about it from another person’s perspective and with another person’s goals in mind. Kafai and Harel demonstrate that this greater “cognitive distance” from the learning process generates conflicts in the consultant’s own knowledge and understanding. Through the process of explaining his ideas to the consultee, the consultant examines his knowledge more closely and is better able to recognize and address any “cognitive conflicts” that he encounters.

An important goal of the social system in AquaMOOSE is to provide a compelling audience that will motivate students to create interesting mathematical artworks. The ISDP project shows that consultation sessions are one way to get students engaged in the
learning task by providing them with a novel audience. In the proposed research below, we have designed consultation into the system in several ways. First, students are encouraged to help each other informally at any time. Second, students who wish to share a problem or accomplishment can participate in an advisory session to get feedback from their peers and the study coordinators. Finally, a formal critique session allows students to give and receive critical feedback on their work. All of these interactions depend on the students’ willingness to help address their peers’ concerns and issues.

Learning By Design (LBD)

Learning by Design is an approach to middle-school science education that uses design challenges to teach students about design practices and skills in addition to more traditional science content (Kolodner, Crismond et al. 1998; Kolodner, Crismond et al. 2003, in press; Kolodner n.d.). One of the main strategies of LBD is to engage students in an iterative cycle of activities that bring the content and practices to the foreground of the students’ attention. Part of this cycle involves students freely exploring the design space, engaging in more structured activities that test and provide feedback on what was learned during exploration, and then reflecting on what they learned and how that knowledge can be effective in other situations.

This process of informal exploration followed by feedback and reflection is an important part of the AquaMOOSE socio-technical system design. Participants in the AquaMOOSE system begin by exploring the supported design space of mathematical art, similar to the ritual practice in LBD of “messing about.” Messing about allows students to formulate their own questions and interests before going deeper to try to understand the underlying concepts. After participants have generated some questions or goals in the AquaMOOSE system, they refine the math they are using and receive feedback from others through activities like advisory sessions, formal critique sessions, or informal interactions. These activities are similar to LBD’s public presentation rituals: poster sessions, pin-up sessions, and gallery walks. Once an artifact has been completed, AquaMOOSE participants provide a written artifact description that contains information about the purpose of the object and the process of creating it. This reflective activity gives the students a chance to carefully think about and explain the knowledge they have gained through designing and creating each of their artifacts. These artifact descriptions can then be referenced as the participants create more artifacts, prompting the students to incorporate past experiences into their new explorations and designs.

MOOSE Crossing

MOOSE Crossing is a text-based online community designed to teach children how to write computer programs (Bruckman 1998; Bruckman and Edwards 1999). MOOSE includes a scripting language that uses natural-language type syntax to allow kids between 8 and 12 years of age to easily access and learn common programming constructs. MOOSE Crossing demonstrates how the Internet can allow people to learn by
creating and sharing personally meaningful objects in a virtual setting. MOOSE users create objects such as pet dogs that follow them around, swimming pools where groups can gather for parties, and shopping malls where others can browse through various creations.

MOOSE Crossing shows that students can become engaged in sharing and discussing their ideas and creations over a computer network. This is another mode of interaction that we have designed into the AquaMOOSE system via discussion boards, rating systems, and instant messaging. Although participants in the proposed after school program will be collocated, they will also be able to use the AquaMOOSE software from home or other settings via an online community. This allows students to continuously share their work and give each other feedback throughout the duration of the program, sustaining the presence and motivational impact of an audience of peers.

**Hypergami**

Hypergami is a software tool created by Mike and Ann Eisenberg to allow students to explore the creation of 3D polyhedral models (Eisenberg and Nishioka 1997). The software generates 2D nets of 3D polyhedral structures, which can then be printed out on paper and folded back into the 3D model. The software supports manipulating and decorating the 2D nets with standard paint tools as well as a programming language that is similar to Logo. Hypergami also supports the creation of “ori-hedra,” or sets of 3D polyhedral models that can be combined to create more complex works of mathematical art.

One of the underlying themes in the Hypergami project is to support students learning how to visualize mathematics, “It is our firm belief that – consistent with the folklore – much of mathematical thinking is indeed distinctly visual in nature, and that mathematical education would therefore profit from a great emphasis on activities that strengthen and exercise visual and spatial reasoning (Eisenberg and Eisenberg 1998).” Visual and spatial skills are especially important when working with more complex mathematics like three-dimensional structures.

In the AquaMOOSE system, participants construct 3D artifacts that combine math equations with artistic goals. While it is not clear exactly how to directly improve visual and spatial ability in students, we recognize the value of those skills for creating interesting mathematical artworks. We have designed the AquaMOOSE software to help students visualize and understand the math they are using, including features like multiple points of view and an animated axis reference tool.

**Logo**

In the LOGO environment, users can write programs to control a cursor on the computer screen. The process of moving the cursor (the “turtle”) around on the screen generates
drawings that are referred to as “turtle graphics.” For example, a simple LOGO program might consist of the commands, “pen down; move forward 100.” This would draw a straight line 100 units on the screen.

This process resembles the way math trails are created in the AquaMOOSE environment. People who see demonstrations of the AquaMOOSE software often refer to the trails as “LOGO in 3D.” However, we incorporated a different strategy in our design of the AquaMOOSE math trails. While the math in Logo is done from a first-person perspective to facilitate body-syntonic understanding of math (Papert 1980), the math in AquaMOOSE is designed to look more like the math students see in school in order to facilitate transfer.

LOGO has been used in many different forms and contexts over the last two decades, but each instantiation focuses primarily on allowing people to use computer technology to create artifacts that leverage their personal interests while providing them with greater insight and understanding of the underlying mathematical constructs. This is the approach we have taken in designing the AquaMOOSE socio-technical system: to provide students with an interesting way to create personally meaningful artifacts while gaining a better understanding and appreciation of math and art.
Research Proposal

In this research, our primary goal is to explore how students’ interest in one area can be leveraged to increase their interest in another area. We have designed a socio-technical system that provides users with the necessary tools to create visually appealing mathematical artifacts. While all students are expected to show some increased interest in both math and art, we hypothesize that students who enter the program with high prior interest in either math or art will show greater increase than students who do not have high prior interest in either area. The secondary goal of this research is to describe the trajectories participants take when creating their portfolios and to compare the trajectories of students with various levels of prior interest in math and art. Table 1 lists the questions we plan to inform through this research.

- How can we use construction in an engaging artistic domain to improve learner interest in academic subjects, specifically mathematics?
- What different trajectories do learners utilize in a constructionist learning environment that invites them to learn an academic subject by engaging in an artistic endeavor, and how do differences in prior interests affect those trajectories?

Table 1: Research Questions

To address these questions, I plan to conduct an after-school program for approximately 15-20 interested, self-selected students at a local high school. The program consists of 12 sessions that meet once a week for one and a half hours. Our goal for the students during the program is to create a portfolio of mathematical art that is presented to friends and family during a public exhibition at the end of the program. In addition to the scheduled program sessions, students can use the software to continue their work from other school labs or from home. I plan to collect and analyze interest surveys, questionnaires, video recordings, observations, interviews, and log files to describe changes in student interest levels as well as each student’s learning process.

We hypothesize that students who enter the program with higher interest in one of math or art will show significantly greater increase in their interest for the other subject, demonstrating the viability of synergistic combinations of academic subjects for improving student interest. We also hypothesize that students who succeed in creating artifacts that meet their goals will show more improvement in their interest levels and that students with different prior interests will utilize different processes while building their portfolios. These hypotheses are listed in Table 2.
Most students’ interest level in both math and art will improve (except those that are already high). The magnitude of this change is expected to be small but significant.

A larger magnitude improvement in interest level will be seen in students who start off with much higher interest in one of (math/art) than the other. The lower area will improve more, because the environment helps the student leverage interest in one area to build interest in the other.

Improvement in interest level will be positively correlated with improvement in portfolio scores and content tests (expertise).

Students with different prior interests in either art or math will approach the program differently and follow different learning trajectories while creating their portfolios.

To address the fourth hypothesis, we will be collecting video data and log file data to describe participants’ process of creation in the environment. One common learning trajectory we have observed while studying the AquaMOOSE software begins with a chaotic stage of random trial and error, followed by the formation of an aesthetic goal, refinement of the math, and eventually resulting in the creation of a personally meaningful artifact (see Figure 2). In formal and informal testing of the AquaMOOSE environment, however, we have observed users approaching the software from different angles. One type of user, who is generally interested in mathematics, tries out all the supported functions, and then slowly tries to understand how manipulating each aspect of the functions changes the look of the resulting artifact. Another category of user types in random conglomerates of math functions until he creates something that looks marginally interesting, and then proceeds to refine the math to make the resulting trail look more appealing. This research aims to further describe these different approaches.

<table>
<thead>
<tr>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form aesthetic goal</td>
</tr>
<tr>
<td>Refine math equations</td>
</tr>
<tr>
<td>Receive feedback</td>
</tr>
<tr>
<td>Complete artifact</td>
</tr>
</tbody>
</table>

**Figure 2:** Typical creation process in the AquaMOOSE system
Study Design

Students from the entire school will be invited to attend a demo session, but math and art classes will be targeted specifically. Students who are interested in participating after attending the demo session will complete an application form that will be used to determine the participants. Based on their responses on the application form, students will be divided into four groups: high math interest, high art interest, high math and art interest, and low math and art interest. Representatives will be selected from each of the four categories via purposeful sampling.

Most study sessions will begin with a 15-30 minute lesson on a particular topic followed by an hour of exploration and challenges. The planned schedule for the program is shown in Table 3. To provide the students with a more structured alternative during the last hour of each session (rather than working on their portfolio), we are also planning fun challenges that are relevant to each topic. Example challenges might be: “Add a 4<sup>th</sup> leaf to this 3-leaf clover” or “Create equivalent 2D spirals in the xy, xz, and yz Cartesian planes.”

The first session will begin with an overview of the program schedule and an introduction to the AquaMOOSE software. This will include a tutorial on navigating the environment as well as creating the basic building blocks of the environment: mathematical trails. Samples will be provided to give the participants interesting starting points for their exploration. Study coordinators will attempt to provide more examples and ideas throughout the study based on participants’ needs.

<table>
<thead>
<tr>
<th>Session</th>
<th>Software introduction</th>
</tr>
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</table>
| Session 2 | Circles and Leaves - Parametric equations  
                        Challenge: Given a 3-leaf clover, add a fourth leaf |
| Session 3 | Making Spheres - Polar coordinate space  
                        Challenge: TBD |
| Session 4 | Still Life - Real-world objects                        |
| Session 5 | Color and Animation - Trail properties                 |
| Session 6 | Critiques / Pin-up session                              |
| Session 7 | Morphing - Trail sequences / Introduce exhibition criteria  
                        Challenge: Create a morph from caterpillar to cocoon to butterfly |
| Session 8 | Scene Construction - Sets of connected trails  
                        Challenge: Build a snowman |
| Session 9 | Free session / Work on portfolio                        |
| Session 10 | Free session / Work on portfolio                        |
| Session 11 | Free session / Work on portfolio                        |
| Session 12 | Public exhibition                                        |

**Table 3:** Study schedule
Sessions 2 through 5, 7, and 8 will begin with a short instruction and demonstration period covering a particular topic. Each session is given both an artistic title and a mathematical title to add further emphasis to the combination of math and art in the environment. The first two of these sessions cover the core math concepts used in the AquaMOOSE software: parametric equations and polar coordinate space. Without some understanding of these concepts, participants will be very limited in what they can create. Sessions 4 and 5 will focus on creating more artistic effects using math trails. Sessions 7 and 8 introduce more complex ways to combine multiple math trails by using trail sequences and sets of connected trails. Sessions 9, 10, and 11 will be devoted entirely to free exploration and participants preparation of their portfolios for the final exhibition, unless advisory sessions are requested.

The final session of the study will be an extended exhibition where each participant has an allotted amount of time to present and explain his or her portfolio. Each object will be accompanied by a written description that follows a basic format provided by the study coordinators, as described below. This session will be videotaped and analyzed to provide further insight into the students’ learning process. Students will be encouraged to invite friends and family to join the other students, teachers, and study coordinators at the exhibition. Participants may choose to have their exhibits and written descriptions displayed on a website hosted at Georgia Tech.

Throughout the study, participants will engage in verbal presentations of their work. All of these presentations are designed to provide rich qualitative data that will help us understand the participants’ learning processes. If a student has a problem with a particular topic or creates something particularly interesting, the study coordinators may call an “advisory session” where the participant can present the problem or accomplishment and receive feedback and suggestions from the rest of the group. The study will also contain one formal critique session where students present their work to the other participants and receive critical feedback. These verbal presentations are intended to help prepare the students for the final presentation of their portfolios, when friends and family will be invited to hear participants describe their works of mathematical art.

Reflection is a critical component of the learning process (Schon 1987; Kolodner 1997). To help support the reflection process, participants will complete a written document to accompany each of their finalized exhibits, similar to the descriptions that appear next to works of art in a museum. This document will contain information describing the process of creating the artifact, as well as the student’s thoughts and feelings about the artifact after its completion. These documents will also be displayed with the corresponding objects on the resulting workshop website. The artifact descriptions will contain the following information:

- Participant’s goal(s) for this artifact
- Math used to create the artifact
- What they’re proud of about this artifact
- What was most fun about this artifact
Data Collection and Analysis

Prior to starting the program, participants will be interviewed to gain a qualitative understanding of their interest level in math and art. They will be given a pre-test on relevant mathematical concepts: trigonometry, parametric equations, and coordinate spaces. They will also complete a survey regarding math and art interest. Halfway through the study, participants will critique each other’s progress in a pin-up session, where each student presents his or her work to the group and gets critical feedback. These sessions provide direction and motivation to the students in addition to rich information about the students’ underlying thought processes that will be used to reconstruct their learning trajectories. After completing the program, participants will again be interviewed, and will complete a post-test on the same mathematical concepts described above. Surveys regarding interest in math and art will again be administered to determine changes. Portfolio scoring, completed by multiple independent raters, will be used to describe both the artistic and mathematical components of participants’ artifacts. Changes in test scores for mathematical understanding will be used to illustrate basic conceptual learning. Changes in artifact complexity, as indicated by the portfolio scores, will also be used to indicate changes in expertise.

Video data captured throughout the study will help reconstruct participants’ learning process and will provide support for changes in interest level as indicated by the surveys described above. Critiques during the pin-up session, advisory sessions, and some independent work sessions will be videotaped to provide further insight into participants’ learning processes. One or more researchers will be in charge of a small video recorder during each session. These researchers will determine which participants to record based on the participant’s activities. Table 4 describes these data and the times during the program when each will be collected.

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Before</th>
<th>During</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interviews</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Math and art attitudinal inventories</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Content test</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Log file data</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Video recordings</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Portfolios of artifacts</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Written descriptions of artifacts</td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

Table 4: Data Collection
Log files will track all aspects of participants’ software use so that sessions can be easily recreated for further analysis. This recreation process will allow us to analyze not only the final artifacts, but also the steps taken to create them. Again, this data will offer more description of how students progress through the creation process. By looking at the intermediate artifacts and the sequence of math equations participants use, we will be able to map each artifact onto the learning trajectory template described above (see Figure 2). Some artifacts may not reach completion. Others may be created through a different trajectory or process. Video recordings will be used to support log file analysis in understanding how participants create each of their artifacts.

Participants will be grouped in one of four categories based on their pre-study interest surveys: high interest in both math and art, high interest in math but not art, high interest in art but not math, or low interest in both math and art. Changes in interest level within these groups will be used to demonstrate the effectiveness of combining these two topics. Changes in portfolio ratings will also be compared to changes in participant interest levels, providing an indication of how students’ performance impacts their interest. Interview data will be used to convey participants’ subjective opinions about their interest in math and art and how the program affected those interests.

Table 6 below summarizes the hypotheses for this study, what data will be collected relative to each hypothesis, how that data will be analyzed, and the expected results and contributions of this research.
<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Relevant Data</th>
<th>Analysis</th>
<th>Expected Result</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Most students’ interest level in both math and art will improve.</td>
<td>- Recorded sessions - Interviews - Interest surveys</td>
<td>Qualitative data will be analyzed to demonstrate changes in student interests. Interest survey scores collected before and after the program will be compared.</td>
<td>Students will show positive change in interest toward math and art.</td>
<td>Synergy between math and art allows students to leverage areas of high interest to improve interest in other areas.</td>
</tr>
<tr>
<td>A larger magnitude improvement in interest level will be seen in students who start off with much higher interest in one of (math/art) than the other.</td>
<td>- Recorded sessions - Artifact portfolios - Content test scores - Interest surveys</td>
<td>Portfolio scores and content test scores will be compared with changes in interest levels.</td>
<td>Students with higher interest in one of the two areas show greater increase in interest toward the lower area.</td>
<td></td>
</tr>
<tr>
<td>Improvement in interest level will be positively correlated with improvement in portfolio scores and content tests.</td>
<td>- Recorded sessions - Artifact portfolios - Content test scores - Interest surveys</td>
<td>Qualitative data sources will be used to describe each student’s learning process.</td>
<td>Different learning trajectories will be described. Comparisons between math-inclined and art-inclined students will be highlighted.</td>
<td>Describe different usage patterns in a constructionist learning environment, specifically those of art-inclined students versus math-inclined students.</td>
</tr>
<tr>
<td>Students with strong prior interests in either art or math will approach the program differently and follow different learning trajectories while creating their portfolios.</td>
<td>- Recorded sessions - Artifact portfolios - Software log files - Interviews</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Study summary by hypothesis
Research Summary

In this research, I propose to instantiate a socio-technical system designed to explore the benefits of combining math and art on student interest levels. The AquaMOOSE socio-technical system I propose is a constructionist learning environment where participants create portfolios of personally meaningful mathematical artworks. In addition to measuring the impact of this system on student interests, I propose to qualitatively describe similarities and differences in the way students with different backgrounds learn in this constructionist environment.

Expected Contributions

This research addresses two issues: how to improve student interest in academic subjects, and how different background interests affect student learning in a constructionist environment. The expected contributions of this research are:

- To demonstrate that combining math and art in a constructionist learning environment positively impacts student interest, especially those students who enter the program with higher interest in one of math or art, and
- To describe how students learn in a constructionist environment and highlight how different prior interests affect the students’ learning trajectories.

Schedule for Thesis Completion

Sept 2003 – Thesis proposal
Sept-Dec 2003 – Proposed study
Jan-Sept 2004 – Data analysis and writing
Sept 2004 – Defense of research
Oct 2004 – Thesis submission
Appendix A: Development of AquaMOOSE 3D

"In school math, 'analytic geometry' has become synonymous with the representation of curves by equations. As a result every educated person vaguely remembers that y=x^2 is the equation of a parabola. And although most parents have very little idea of why anyone should know this, they become indignant when their children do not. They assume that there must be a profound and objective reason known to those who better understand these things. Ironically, their mathophobia keeps most people from trying to examine those reasons more deeply and thus places them at the mercy of the self-appointed math specialists. Very few people ever suspect that the reason for what is included and what is not included in school math might be as crudely technological as the ease of production of parabolas with pencils! This is what could change most profoundly in a computer-rich world: The range of easily produced mathematical constructs will be vastly expanded.” (Papert 1980)

Work on the AquaMOOSE project began in 1997. Since then, we have completed three separate prototypes of our software and conducted many informal and formal user studies. Our first prototype, NetFlyer, was built using the OpenGL language. The second prototype was built with Randy Pausch’s Alice software. Our current implementation is an MFC application programmed in Visual C++ using a freeware rendering package called Genesis 3D. The current implementation has gone through many revisions over the last several years based primarily on user feedback.

NetFlyer

Our first prototype, called "Net Flyer," was a simple two-person game similar to the American basketball game 'HORSE' (see Figure 3), created in OpenGL. Two players play at one terminal. While one turns his or her back, the first player selects an equation. The second player views the graph of the equation (metaphorically, the path of a flying disk) and tries to guess the original equation. An unsuccessful guess earns a letter in the word "FLYER." The first player to complete the word "FLYER" loses. From this early prototype, we gained insight into the kinds of visualization supports we would need, such as the ability to switch cameras to make use of multiple views of the 3D world. In informal testing, users noted that the equations were often aesthetically pleasing. They also observed that it was important to keep equations simple if their opponent had any hope of guessing them. The game was more fun in a cooperative mode where you try to provide your partner with an appropriate challenge, rather than a competitive one where you try to stump him or her.
Alice Prototype

The original NetFlyer prototype had a very primitive interface. Since the goals of the project include leveraging the aesthetic appeal of 3D graphics, we knew we needed to improve the look of the software. To move toward that goal, our next prototype was developed using Randy Pausch's "Alice" software (Conway, Audia et al. 2000). In that version, we created a 3D avatar world with the same flying disk game theme from NetFlyer (see Figure 4).

Compared to our first prototype, this new avatar world seemed much more fun, at least on the surface. The visual look and feel bear a resemblance to game-like environments, and the graphics are attractive. However, it was also much harder to understand the mathematics of motion in 3D. In our first OpenGL prototype, the user's perspective was fixed, making the orientation of the coordinate system consistent and clear. With the moving point of view of a first person system, all the math became harder to understand visually. This observation informed our addition of visualization supports like overlaid axes, grid tools, and multiple viewpoints in our next prototype.

The Alice software proved to be an excellent prototyping tool. However, during our exploration of the Alice prototype, we noted a limitation to our theme: the disk moves in three dimensions, but the user's avatar is stuck in a plane. We also recognized that in order to keep the game-like feel of a 3D world but make mathematical understanding easier again, we needed finer control of the user interface.
To better make use of free movement in all three dimensions, we switched to an underwater theme—fish swim freely in three dimensions. Since this work is partly inspired by our prior research on end-user programmable textual worlds in the MOOSE Crossing project (Bruckman 1998; Bruckman and Edwards 1999), the new project was jokingly dubbed "AquaMOOSE 3D." While the name was proposed as a joke, it stuck. At this stage, we next moved to a development platform that gives us more control over the user interface: Microsoft Visual C++ and the freeware Genesis 3D rendering engine (Eclipse 1998).

Motion in AquaMOOSE can be specified mathematically, using parametric equations. For example, swimming in a sine wave in $x$, a cosine in $y$, and a constant in $z$ creates a spiral. Cartesian, cylindrical, and spherical coordinate systems are supported.
A simple template scaffolds (Collins, Brown et al. 1989; Guzdial 1995) the process of entering in mathematical moves (see Figure 5). Each time a math move is executed, the avatar moves along the programmed function, leaving a trail behind. The trail is an important artifact; it provides the users with a visualization tool for instant feedback and a starting point for conversation (see Figure 6).

In many ways, this resembles the Logo programming language (Papert 1980) in three dimensions. However, there are some important differences. While the math in Logo is done from a first-person perspective to facilitate body-syntonic understanding of math (Papert 1980), the math in AquaMOOSE is designed to look more like the math students see in the classroom in order to facilitate transfer.

We also provide one prototype game for students to play in the AquaMOOSE environment: a ring game. Students are presented with a set of rings in the water, and are challenged to try to swim through as many as possible with one mathematical function. This simple game resembles a 3D version of the successful Green Globs software by Sharon Dugdale (Dugdale 1982). In user studies, this game turned out to be too difficult for students to use, and has therefore been de-emphasized in the current version of the software.

**AquaMOOSE 3D – Current Version**

Based on several rounds of informal and formal evaluation, we realized that the most appealing aspect of the AquaMOOSE software is the visualization potential of the math trails in 3D. Users of all ages have been intrigued by the aesthetic quality of these mathematical artworks. Continuing in our iterative design process, we used this feedback to direct our next version of the software. Our goal for the new version is to incorporate features that promote connections between math and art, allowing users more control over the aesthetic characteristics of their creations.
For the current version of the AquaMOOSE software, we have developed three major features that provide users with more possibilities for creating and customizing their artifacts. Trail animation properties, trail sequences, and sets of connected trails allow greater flexibility and diversity in the types of objects that can be created using the AquaMOOSE software.

Figure 7: Customized trail properties

The current version of the software provides many more options for customizing the look of any math trail (see Figure 7). Trails can be drawn using a solid color or a blend of two different colors. When trails use two colors, the blend can be animated by either pulsing between the two colors or by shifting the colors down the trail in a sine wave pattern. The rate at which this animation occurs can also be controlled. In addition to colors and animation, the width of the trail can be changed to create thick or thin trails. Users can also change the number of segments calculated for a particular trail. Although increasing the number of segments also increases the amount of necessary computation, it allows the user to control the appearance of trails in interesting ways. Computational artifacts can be used to create effects like “barbed wire” or moiré patterns.
Another new feature in the current version of the software is called trail sequences. Trail sequences allow the user to create a trail that morphs through a series of parametric equations (see Figure 8). This morphing can be synchronized to outside events such as audio streams or video streams, similar to visualization tools seen in popular MP3 players. Two values for each set of equations control the morph time and duration for that entry. The morph time determines how long it takes the trail to morph to the appropriate equations. The duration of the entry is the amount of time before the sequence moves on to the next set of equations.

Finally, the new version of AquaMOOSE includes the ability to connect multiple trails to form a more complex artifact. Once a trail has been created, the user can select that trail...
and choose to create one or more “child” trails that are attached to the original trail. Child trails originate at a selected point along the parent trail. An example of a set of connected trails might be a mathematical fish that has a body, fins, and eyes (see Figure 9). The body would be one trail, while each fin and eye would be a child trail that is connected to the body trail at the appropriate point. Another example might be writing a person’s name or other text message using connected trails.
Appendix B: Formative Evaluation of AquaMOOSE 3D

We have completed several rounds of formative evaluation during the last several years. Details about all of the AquaMOOSE formative evaluation trials are shown in Table 7.

<table>
<thead>
<tr>
<th>Date</th>
<th>Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter 2000</td>
<td>Small group of college freshmen</td>
</tr>
<tr>
<td>Spring 2001</td>
<td>Advanced math class from Atwood Private School</td>
</tr>
<tr>
<td>Spring 2001</td>
<td>Students from Georgia Tech</td>
</tr>
<tr>
<td>Spring 2001</td>
<td>Six local high school math teachers</td>
</tr>
<tr>
<td>Summer 2001</td>
<td>Georgia Governor’s Honors Program</td>
</tr>
<tr>
<td>Fall 2001</td>
<td>Math club at Brooks High School (public)</td>
</tr>
<tr>
<td>Fall 2001</td>
<td>Advanced math class from Atwood Private School</td>
</tr>
<tr>
<td>Spring 2002</td>
<td>Two precalculus classes at Brooks High School</td>
</tr>
</tbody>
</table>

Table 7: Dates and participants of AquaMOOSE formative evaluation trials

A small group of college freshman used the AquaMOOSE software in a laboratory session in late 2000. We observed that co-location was an important aspect of the learning process. Students looked over one another’s shoulders sharing excitement about their creations and trading mathematical strategies.

In Spring 2001, we invited a small advanced math class from a local private high school, which we will call Atwood Private School, to evaluate the software in our laboratory. One student in that class, Joel, completed an entire ring course in about 15 minutes, whereas his classmates and teacher took nearly an hour. On approaching the system, Joel immediately understood exactly what to do, and solved the first segment of the first puzzle in seconds. We were surprised to see such a large range in achievement within an honors class. Based on conversations with Joel and his teacher, we believe his success is attributable to his unusually well developed skill in both mathematics and visualization. Joel was energized by his experience, and continued to explore the system avidly while his classmates finished their work. We are continuing to refine the system’s visualization supports to try to make it as immediately accessible to others as it was to Joel.

We then conducted several rounds of formative evaluation with other students at Georgia Tech. The results from these multiple rounds of formative evaluation have helped shape the current system design and user interface.

In addition to our testing with students, we invited six local math teachers to talk with us about AquaMOOSE and how it might be used in their classes. Based on those conversations, we developed a better idea of how to integrate the application with a pre-calculus curriculum.

During the summer of 2001, we conducted our largest formative evaluation with 105 high school math students at the Georgia Governor’s Honors Program. This study is discussed below in more detail (see “Governor’s Honors Program Study” below).
Our next trial was in fall 2001 with an after school math club at a local public high school, which we will call Brooks High School. Brooks High School is in a less affluent suburb of Atlanta. The majority of the students qualify for free or reduced lunch, which is an indicator of socio-economic status. The students at Brooks also enjoyed playing with our software, but were slightly distracted by the informal atmosphere of the math club. This trial was mainly to demonstrate the software to the math teachers at the school in preparation for a future study we will be conducting in some of the pre-calculus classes there.

Also in fall 2001, we conducted a trial with this year’s advanced math class from Atwood Private School. The private school students were able to carry on involved conversations with us about the concepts involved in the AquaMOOSE software. It was especially interesting to hear that this particular class had already covered one of the most difficult concepts in our software, 3D polar coordinate space (or spherical polar coordinates).

Our most recent evaluation was a comparison class study at a suburban high school in early 2002, where we compared the AquaMOOSE intervention to instruction using the traditional curriculum. The comparison class learned about polar coordinate space using the standard curriculum guide, while the experimental class used a curriculum that we designed around the AquaMOOSE software. This study is also discussed in more detail below (see BHS Comparison Class Study below).

**Governor’s Honors Program Study**

In summer 2001, we conducted a study at the Georgia Governor’s Honors Program (GHP) in Valdosta, Georgia. Each year, rising juniors and seniors from Georgia high schools are nominated to attend a six-week summer camp to explore a particular academic area. One of the subject areas offered at GHP is mathematics. There were 105 students majoring in math at GHP that summer. Those students were the subjects for our study. We installed the AquaMOOSE software on 31 computers in the GHP labs that the students used for classes. The students also had access to these labs when classes were not being taught. One of the mandates for the GHP is to enrich students’ learning experience beyond what they encounter in the standard curriculum. That aligns very well with our fundamental research goals, making the GHP an ideal setting for this trial.

The study began with a 45-minute demonstration of AquaMOOSE 3D. After that, the students were allowed to log into AquaMOOSE freely. After the introduction, they were not required to use AquaMOOSE at any time. We were more interested in collecting data about how the students used AquaMOOSE during their free time than during required time. There were many other projects and activities at GHP that the students could participate in during their free time. Playing with AquaMOOSE was only one option for the students. All of the data presented in this paper was collected from students who voluntarily used AquaMOOSE throughout the summer.
Students had access to the software in a computer lab setting. Students were able to talk to one another and discuss their progress while using the computers, but generally used the software individually. We collected log files detailing the students’ usage of AquaMOOSE over the next six weeks. We returned to GHP half way through the summer and observed the labs during students’ free time. Our final visit to GHP was at the end of the program, when we collected anonymous surveys from 103 students (2 had left the program during the summer) and conducted interviews with 10 students. The GHP lab administrators and instructors chose the 10 interview subjects who had shown the most interest in AquaMOOSE during the summer. Most of the students who were interviewed found the graphical nature of the program appealing, but wanted more features and more goals to the game. Many of them wanted the fish to be able to jump out of the water or be able to eat other fish. After the study, we did a more in-depth interview with one outstanding student.

**Results and Discussion**

There were 105 math majors at GHP who completed parental consent forms for our study. We asked the students to provide their real name and a pseudonym to be used in the software. Many of the students, however, did not provide their real name. In addition to the 105 math majors at GHP, students from other programs had access to the computer labs and our software. Since some of the math majors did not provide their correct real name when registering for an online account, they were indistinguishable from the students who had not signed consent forms. Due to that problem, we are only able to report on the data from those accounts that are verifiable as having provided parental consent for our study. Of the 105 math students, 63 created verifiable accounts. Of those 63, 37 students (59%) performed at least one mathematical movement. The total number of mathematical movements performed by all 63 students was 962. One student, Mark, was responsible for 340 of those moves. We discuss Mark’s experience in greater detail below.

- Were there things that you wanted to do in the environment that you couldn’t do?
- What could we do to improve AquaMOOSE 3D?
- Would you be likely to play with AquaMOOSE 3D if it weren’t for a school assignment?
- What were your favorite and least favorite aspects of AquaMOOSE 3D?

**Table 8: Sample questions from the survey**

At the end of the six-week study, we distributed anonymous surveys to the students. The majority of the questions on the survey were open-ended discussion questions. See Table 8 for some examples of the questions we asked in the survey. Two reviewers analyzed the surveys. During that analysis, the reviewers noted whether each survey mentioned particular topics of interest. The six topics we chose to explore were negative aesthetics, positive aesthetics, game goals, violence, community, and competition. Some examples
of comments for each of the categories are shown in Table 9. Our average inter-rater reliability was 84%.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Sample Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative Aesthetics</td>
<td>“[I wanted to] swim in a larger environment with more space.”</td>
</tr>
<tr>
<td>Positive Aesthetics</td>
<td>“The environment and the trails were some of the best parts.”</td>
</tr>
<tr>
<td>Goals to the game</td>
<td>“I think I might play around with it for a little bit but not regularly because I don’t feel like there’s any goal to it or clear way to win.”</td>
</tr>
<tr>
<td>Violence</td>
<td>“[I wanted to] eat the smaller fish.”</td>
</tr>
<tr>
<td>Community</td>
<td>“I would play… longer if I could interact and share stuff with my friends.”</td>
</tr>
<tr>
<td>Competition</td>
<td>“[I wanted to] race with others.”</td>
</tr>
</tbody>
</table>

Table 9: Sample comments from surveys

Out of the 103 students, the number who mentioned aesthetics in a negative manner (56) was almost identical to the number who mentioned positive aspects of the environment (55). Many students requested more structured goals in the environment (18), more violence (14), or more community involvement (12). A few of the students specifically requested that competition be a more integral part of the software (5). These results are shown in Table 10.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Occurrences (N=103)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative Aesthetics</td>
<td>56 (54%)</td>
</tr>
<tr>
<td>Positive Aesthetics</td>
<td>55 (53%)</td>
</tr>
<tr>
<td>Goals to the Game</td>
<td>18 (17%)</td>
</tr>
<tr>
<td>Violence</td>
<td>14 (14%)</td>
</tr>
<tr>
<td>Community</td>
<td>12 (12%)</td>
</tr>
<tr>
<td>Competition</td>
<td>5 (5%)</td>
</tr>
</tbody>
</table>

Table 10: Anonymous survey results.

**Usability of 3D**
As we expected, the majority of the students mentioned that the 3D graphics in AquaMOOSE were appealing. They enjoyed editing their fish avatars, looking at the math trails they created, and exploring the various 3D worlds that were provided with the software. One of our basic premises for this project is leveraging the appeal of 3D games to improve students’ motivation to learn mathematics.

While they liked the visual look of AquaMOOSE’s graphical world, many students experienced problems navigating in it. Some of the problems involved the navigation
controls, while others involved representational problems about the virtual world. The size of the environment that we should provide in AquaMOOSE is a difficult design problem. Overall, larger worlds are more popular with the students because they offer more opportunities for exploration. However, there is an important difference between providing interesting places for students to explore and providing large open places for them to explore mathematics. Some of the worlds we included during the GHP study contained interesting “hidden” features that the students could discover through exploration. However, none of the large open spaces we provided were big enough for many of the mathematical moves the students attempted.

One of the most prominent comments we received from the students was that they did not like colliding with walls (referred to as “bonking”) because their math moves did not fit in the world. Since the GHP study, we have constructed a few very large worlds to help alleviate some of those space concerns. Simply creating larger worlds does not solve the problem, though. In the larger worlds, moving an avatar from one side of the world to the other with our standard mouse and keyboard navigation controls takes quite some time. The key issue for this problem is the scale factor between the mathematics and the world geometry. We are exploring ways to allow the scale of math functions to be controlled by the students.

Another important aspect that we have learned from this study is that visualization of math trails is interesting to the students. The students had not been exposed to this type of mathematical visualization before, and described wanting more tools to help understand the graphical representations. Many students wanted more cameras to move around in the environment. They also wanted more control over what was shown in the cameras that we provided. We are working to implement zooming and panning functions on all cameras in AquaMOOSE 3D.

Mark’s Experience
Of the students at GHP, the most enthusiastic AquaMOOSE user was Mark. His experiences were by no means typical. We chose to examine his success story in detail to learn what went right. We hope to use those insights to guide future iterative design.

Mark is 17 years old, and is a rising senior at a high school in a small Georgia town. Mark’s creations in AquaMOOSE drew the attention of his instructors and peers at GHP. His trails consisted of complex equations that he used to represent surface-like structures. Not only were they mathematically sophisticated, but also aesthetically pleasing (see Figures 10-12). We chose to study Mark’s experience in detail to explore what factors aided his success. Why did he become more engaged than other students?

To understand Mark’s experiences, we have examined logs of his AquaMOOSE usage, interviewed him during GHP, and interviewed him again after GHP. Mark’s favorite activities include playing guitar, writing poetry and short stories, and playing around with computer programs. Mark was nominated for GHP in both math and English. GHP offers a program in music as well, but Mark’s high school does not nominate students for that major. He decided to attend GHP this past summer partly because he knew it would be
good for his academic career, but also because he thought it would be an interesting social experience.

Since Mark attended GHP as a math major, one might guess that his favorite subject in school is mathematics. However, it turns out that is not actually the case. Mark says that his favorite subject changes very often, usually rotating between creative writing, music, and math. “I like all of them a lot, and it just depends on whatever strikes me… what I’m curious about that second,” Mark says. Since AquaMOOSE is designed to take advantage of a synergistic combination of math and art, it is not surprising that the most enthusiastic user is equally interested in humanities and mathematics.

Mark thinks he would like to become a computer programmer when he goes on to college next year. However, he feels that his lack of experience with programming during high school will cause him to be behind other students at college. If that proves to be the case, he is considering an engineering career as an alternative. Regardless of which major he chooses, he intends to get a minor in music, which he will use as a back-up career. He entertains thoughts of teaching music at some point during his life.

On arriving at GHP, Mark was surprised by the intelligence of his peers. Entering an environment where the majority of the students around you are the brightest at their respective high schools can be an intimidating experience.

There were many other activities going on at GHP that took away from the students’ free time. As Mark pointed out in our interviews, GHP is a social experience as well as an academic one. Many of the students spent a great deal of time playing sports, attending dances, or just hanging out with their new friends. Mark balanced out those activities with his exploration of AquaMOOSE 3D. “There wasn’t actually that much stuff that I wanted to go to that I missed,” Mark says.

Mark used AquaMOOSE mostly during his free time. After checking his email, he would play around with AquaMOOSE for a while. His typical sessions were anywhere from 15 minutes to an hour. There were other things on the computer that Mark could have used instead of AquaMOOSE, but none of them held his attention for very long. Even a text-
based role-playing game that he had seen did not provide him with the entertainment that AquaMOOSE did. Mark usually enjoys playing video games, but is disappointed because many 3D games in particular contain too much violence and non-intuitive interfaces to the 3D environment. During the six-week period of GHP, Mark used AquaMOOSE for over 10 hours across 16 sessions. Even Mark’s friends and teachers at GHP noticed his high level of involvement with the software. The teachers and students alike were impressed with Mark’s creations in AquaMOOSE.

Mark’s initial reaction to AquaMOOSE was frustration about the equation interface. The template for parametric equations in AquaMOOSE provides an area to enter the \( x(t) \), \( y(t) \), and \( z(t) \) functions. Many of the students, like Mark, entered equations in the form \( f(x) \), \( f(y) \), and \( f(z) \), using \( x \), \( y \), and \( z \) as the variables instead of \( t \). In typical math classes, students most commonly see equations expressed in terms of \( x \), \( y \), and \( z \). Most students have limited or no exposure to parametric equations expressed in terms of \( t \). Since the software evaluated those other variables (\( x \), \( y \), and \( z \)) to zero, most of Mark’s initial attempts at math moves produced no movement at all. The version of AquaMOOSE used at GHP was still in an early prototype stage, and did not provide the appropriate feedback to help correct such common mistakes. After a couple of days, though, Mark realized what the equations were expected to look like, and began experimenting with AquaMOOSE again. Mark spent most of his time in AquaMOOSE doing free exploration of complex math equations as opposed to playing with the ring game.

Mark’s first exploration of AquaMOOSE involved playing around with the sample spiral move that is included with the software. The spiral consists of the equations:

\[
\begin{align*}
    x(t) &= \sin(t) \\
    y(t) &= \cos(t) \\
    z(t) &= t
\end{align*}
\]

These simple equations produce a visually appealing spiral in the 3D environment that was intended to spark the interest of students. After Mark played around with the spiral for a while, he moved on to more complex representations.

This second phase of his exploration resulted in what he describes as “a plane with a spiral coming out of it” (see Figure 10). Another one of Mark’s creations produced a trail that looks like a tunnel, except it has “barbed wire” sticking into the tunnel. Mark continued to experiment with various math functions. He added complexity to the equations in various ways. Some of his moves involved combining equations, like \( t\sin(t) \) instead of just \( \sin(t) \). Some more examples of his work are shown in Figures 11 and 12.

Eventually, a new goal emerged in Mark’s mind. He wanted to make a math move that formed a sphere in the 3D environment. He made several attempts at the sphere, but was unable to complete his goal. After the GHP program ended, Mark indicated that he was still thinking about how he could have made the sphere, and was disappointed that he didn’t have access to the software any more so that he could test his hypotheses.
In his current high school math classes, Mark does not use computers very often. The main form of technology that he has been exposed to is a graphing calculator. The Texas Instruments graphing calculators are used in many math classes in the United States. Mark feels that AquaMOOSE offers a lot more than the graphing calculator. “Most of the time in math classes you don’t have any way to actually represent 3D graphs. Sure, you can do 2D graphs; that’s what the TI-83 is for… AquaMOOSE gave you the chance to do that stuff and had a user-friendly format where you could move around and leave a trail.”

Mark speculates that teachers would appreciate using AquaMOOSE in the appropriate classes. He also thought that the teachers would probably spend their own free time exploring AquaMOOSE, since the 3D visualization is not supported well in other common tools.

A key question remains to be answered: what math did Mark learn? Based on the artifacts he created and our interviews with him, we know that he was successfully able to move from a goal to its execution. He wanted to create trails with particular shapes and aesthetics, and was able to do so through iterative experimentation. We know that he became increasingly fluent with combining basic mathematical elements to achieve his desired goal. We do not know to what extent that mathematical insight will transfer to other contexts.

Figure 11: Mark’s complex “funnel” move.

\[
\begin{align*}
  x(t) &= .03 \cdot t \cdot t \cdot \cos(40t) \\
  y(t) &= .03 \cdot t \cdot t \cdot \sin(40t) \\
  z(t) &= t
\end{align*}
\]
Figure 12: Mark’s “spiral spiral” move.

\[ x(t) = 0.08 \times 30 \times \cos(40t) + (1.2 \times \sin(0.5t)) \]

\[ y(t) = 0.08 \times 30 \times \sin(40t) + (1 \times \cos(0.5t)) \]

\[ z(t) = t \]

**BHS Comparison Class Study**

BHS is a suburban public high school in central Georgia. The students at BHS are typically less advantaged and a large percentage of them receive free or reduced lunch. The teachers’ expectations of the students and the students’ expectations of themselves tend to be low. For example, the probable valedictorian of this year’s senior class plans to go to a local community college. When asked why she would not consider a four-year college, she expressed doubt about her qualifications.

The subjects for the study came from two pre-calculus classes at BHS. The two classes were roughly the same size (N = 30 and N = 34), and met in the same classroom and computer laboratory settings for the duration of the study. The same teacher taught both of the classes.

The teacher in our study, Kimberly, has been at BHS for 17 years. She has seen the school demographics transition from predominantly Caucasian to predominantly African American and has dealt with the racial tension that has ensued. In several instances, her safety has been threatened on school grounds. In fact, when asked what the most challenging aspect of working at BHS was, she stated “the hallway.” While she feels safe within her classroom, she has been called names, yelled at, and even choked by a student when walking around other parts of the school.

In late January, both of Kimberly’s pre-calculus classes began studying polar coordinate space. This unit was taken from the county’s Curriculum Guide for Advanced Algebra and Trigonometry. The comparison class for our study used the standard curriculum guide and tools while learning about polar coordinate space. The second class used the
AquaMOOSE curriculum and the AquaMOOSE software in addition to the standard tools.

The Curriculum Guide states that the unit on polar coordinate space should last roughly one and a half weeks. The standard method for teaching polar coordinate space is by following Chapter 9, titled “Polar Coordinates; Vectors”, in their textbook. Normal tools for the comparison class consisted of pencil and paper materials as well as hand-held graphing calculators (TI-83, TI-85, etc.). No extra software or technology was used in the comparison class.

Miller et al. note, “Learning outcomes achieved through microworld interaction depend largely on the surrounding instructional activities that structure the way students use and interact with microworlds” (Miller, Lehman et al. 1999). We developed the AquaMOOSE curriculum based on the standard curriculum found in the Curriculum Guide to help provide that structure. In addition to the standard topics, however, the AquaMOOSE curriculum included a review of trigonometric concepts, an introduction to parametric equations, an explanation of 3-dimensional functions, activities involving both cylindrical and spherical polar coordinate space, and a more unstructured lesson that focused mainly on the Ring Game portion of the AquaMOOSE software. The teacher covered parametric equations from the textbook in the comparison class to accommodate the requirements of the AquaMOOSE software.

The AquaMOOSE software was used to demonstrate various concepts during the week and a half unit on polar coordinate space. The software was tightly integrated with the AquaMOOSE curriculum. The software was available in the BHS computer lab for the duration of the study. All data from the students’ use of AquaMOOSE was logged and stored on a server at Georgia Tech.

Method
We began the study by testing the students’ visual ability and attitudes towards mathematics. We used three tests (CS-2, S-2, and VZ-2) from the Kit of Factor Referenced Cognitive Tests (Eckstrom, French et al. 1976), available through ETS, to measure the students’ visual ability. For our attitude survey, we used the Fennema-Sherman Math Attitude Scales (Fennema 1976).

The length of the study was 8 school days. Table 11 below describes the activities of the comparison and experimental classes throughout the study. The first day was devoted to administering the pre-tests of visual ability and math attitudes. The comparison class met all 8 days in their normal classroom. They also took the visual ability and attitudes tests on the first day of the study. On the eighth day, the teacher handed back the content tests and went over the students’ grades with them. We also gave the students a short questionnaire and a post-survey of their math attitudes. After completion of the unit, we conducted interviews with 3 students from the comparison class, 8 students from the experimental class, and the teacher. Three months later, at the end of the school year, we provided an open-ended survey about the content material and the math class in general.
Our curriculum called for the students in the experimental class to have two days of preparatory work in the classroom, followed by three sessions with our software in the computer lab. However, due to scheduling conflicts, we were only able to get two consecutive days in the lab on the fifth and sixth days of the study. The only other day that the computer lab was available was the second day of the study, before the students had any exposure to the content material. That session was used to acquaint the students with the AquaMOOSE software. The third and fourth days were spent learning some of the content material in the classroom. The fifth and sixth days were spent in the computer laboratory. The content test was given on the seventh day.

The content test consisted of three sections. The first section covered material about the basics of polar coordinate space. The second section dealt with graphing polar equations. The third section was on parametric equations and 3D graphs of polar equations. The first two sections would have been covered regardless of our study. However, AquaMOOSE required the introduction of parametric equations in addition to polar coordinate space. The teacher covered parametric equations from the textbook in the comparison class, while the experimental class learned about parametric equations mainly through the use of the AquaMOOSE software.

Our hypotheses for this study deal with a range of factors that might affect the usefulness of AquaMOOSE in the classroom, including visual ability, math attitudes, and prior experience with video games. The following is a complete description of our hypotheses.

1. Students with higher visual ability will be more likely to benefit from using the AquaMOOSE software.
2. Students' attitudes toward math and math learning will be more likely to improve in the experimental class than in the comparison class.
3. Students in the experimental class will report a more positive experience learning these topics.
4. Students in the experimental class will exhibit more motivation in post-interviews about these topics.
5. Students in the experimental class will be more likely to remember this particular unit in the end-year survey.
6. The experimental class will show an improvement in their assessment of their teacher's support for their learning.
7. There will be no changes in the teacher attitude assessment for the comparison class.
8. Students with prior video game interest and experience will benefit more from AquaMOOSE both in math achievement and attitudes.
9. Students in the experimental class who score lowest on the spatial tests may show worse math attitudes in the post-test.
We have no prediction whether the experimental class will perform the same, better, or worse than the comparison class with respect to polar coordinates. Benefits of the software may be offset by wasted time going to and from the computer lab, and reduced teacher control of the learning situation in the computer lab.

The experimental class will perform better than the comparison class on parametric equations and 3D.

**Results**

We used the students’ grades from the first semester of the pre-calculus class as a measure of prior mathematical achievement. The experimental class had slightly higher grades than the comparison class, but the difference was not statistically significant (P > 0.1).

The results from the visual ability tests did not predict benefits from the AquaMOOSE intervention on content test scores or attitudes. There were no significant changes in students’ attitudes about math during the study. The AquaMOOSE intervention had no impact on the students’ performance on the content test or on their attitudes about mathematics. Students from the comparison class scored slightly better on average than students from the experimental class on each of the three sections of the content test. However, none of the differences were statistically significant. The students who indicated prior experience with video games had higher average scores on the content test, but the difference was not significant. The AquaMOOSE intervention did not increase the advantage of having prior video game experience (the students with video game experience in the comparison class did better than the students with video game experience in the experimental class). In short, all but two (#7 and #10) of the hypotheses listed above were not supported.

While the students showed some understanding of the subject material on the content test, our retention survey at the end of the school year indicated that very few of the students remembered anything about the unit on polar coordinates. This lack of retention was true of the students in the experimental class as well as the students in the comparison class. In response to a question on the retention survey about polar coordinate space, one student in the experimental class commented, “After the test I will forget, because it’s not interesting to me.”

Four of the eight students interviewed from the experimental class did not find the use of the AquaMOOSE software helpful in learning the content material. When asked if she enjoyed using the software, one student said, “I mean, I really didn’t understand it overall. It was ok. But like just to do, I wouldn’t do it. Not to just have fun. I didn’t think it was fun. If anything, it confused me even more.” Some of the students did see benefit from using the software, but noted issues about less time on task and confusion in the computer lab.

Student responses in the survey at the end of the year were similar. Some students had a positive impression of the software. One student said that while he did not enjoy AquaMOOSE as a game, he liked being able to visualize the mathematics in a better way. Many of the students expressed negative comments about the AquaMOOSE intervention.
In response to a question about his or her least favorite part of math class, another student responded, “AquaMOOSE was aweful[sic]. I didn’t learn a thing, my mind just got confused and unoriented.”

Discussion
Before the study began, the teacher announced to her classes that one section was going to be using the AquaMOOSE software in the computer lab, and one was going to stay in the classroom. Of course, the students in the comparison class wanted to be the ones to use the software instead. They argued that they “were good at video games” too and should get a chance to use AquaMOOSE. Those students had high expectations of the software, and thought it was unfair that they were not allowed to use it until the study was completed.

Despite this initial motivation to use AquaMOOSE, many students in the experimental class were disappointed with the software. After the study, students in the experimental class commented on the lack of action in AquaMOOSE and the imperfect models and environments that we used. One student, in response to a question about polar coordinates, said, “I don’t remember anything but the ugly little fish.” By telling the students beforehand that they were going to be using software that was game-like in nature, we set the AquaMOOSE software up to compete against commercial video games. As can be seen by the intense competition present in the commercial video game market, the students’ high expectations are difficult to meet. For example, to create the massively multiplayer online role-playing game Asheron’s Call, Turbine Entertainment had a staff of over 30 people working for 4 years (Ragaini 2000). A research prototype made by a few graduate and undergraduate students and one faculty member clearly cannot compete.

AquaMOOSE was designed to allow free exploration of 3D mathematics using primarily parametric equations and trigonometric functions. The math content of the AquaMOOSE software is very different from what is typical in high school math classes. 3D mathematics of any kind is rarely present in high school curriculum guides. The math in AquaMOOSE is not only different from anything the students had seen before, but is more difficult and is not on any of the standardized tests the students are required to take. Navigation in the 3D environment also creates confusion. Despite the use of a navigation scheme commonly used in popular games, many students had trouble controlling their avatar’s movement in the 3D setting.

Like many typical school computer labs, the lab at BHS does not readily support a classroom atmosphere. Several factors, such as broken computers, insufficient space, not enough chairs, scheduling conflicts, and the arrangement of computers into rows facing different directions, contributed to making the computer lab an unproductive setting. Several new research initiatives incorporate the use of wireless handheld devices to support learning in the classroom as an alternative to computer labs (Roschelle and Pea 2002). In reference to the problems associated with computer labs, Hickey et al. state, “that providing computer access in the classroom enhances the teacher’s ability to use the technology-supported curriculum to support meaningful learning (Hickey, Kindfield et al.
Based on those findings, avoiding the computer lab and providing technology within the more accepted learning atmosphere of the classroom would probably increase the impact of the AquaMOOSE intervention. In addition to problems with the computer lab, there were several problems in the classroom itself. During our study, there were several times when students were called out of class to participate in required testing or other school functions. Some students were disruptive during timed tests and surveys, distracting any nearby students. The classroom was generally a hectic and often unpredictable environment.
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